Inspection-Warranty Policy for Weight-Quality in Monopoly

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Abstract

In the final stage of manufacturing some specific products, there is a process where we weigh each product using a scale to mark each weight on the product. However, the scale occasionally becomes un-calibrated, and such inaccuracy of a scale can be detected only by periodical inspection. This study considers two types of inspection policy for a scale, both of which carry out inspection to the scale at scheduled time iT(i=1,2,...). Under Policy I, each product is shipped out immediately after we weigh each product. Under this policy, we have a risk to ship out products with a label or a mark showing incorrect weights and hence we need some warranty for such defective products. Under Policy II, however, each weighed product is not shipped out before we perform the inspection to the scale although we should devote more expense to the weighing process under this policy than under Policy I. We compare Policy I with Policy II from both the consumer's viewpoint and the manufacturer's one through a Stackelberg game formulation to discuss an optimal warranty strategy for the manufacturer.

1 Introduction

In the final stage of manufacturing for some specific products such as chemical products, there is a process in which we weigh each product using a scale with a view to obtaining its exact weight, and then marking each product with its weight. This weighing process is necessary in the situation, e.g., where drums are filled with some specific chemical product so that each drum contains approximately 250 kilograms of the product, and in the final stage, individual drums are weighed to obtain the actual weight of each drum of product.

Such a weighing process is not necessarily emphasized and its associated cost is reduced as much as possible since it does not affect the product quality itself. However, the scale occasionally becomes uncalibrated particularly when the objective product is very heavy or we are very busy in weighing many products within a restricted time. Once the scale becomes uncalibrated, it will produce inaccurate weights for individual products, and hence there is a risk that the products will be shipped out with marks or labels indicating incorrect weights. In this study, when a product with a mark or a label revealing incorrect weight is shipped out, it is referred to as *defective* regardless of its quality. Under real circumstances, such inaccuracy or uncalibrated state of a scale is detected by periodical inspection.

In the cases where the products are expensive or exact weight is a critical factor, the scale will be inspected and found to be normal prior to each shipment. In other cases, however, each lot of products may be shipped out immediately after they are weighed without the scale being inspected. This is because of cost reduction for this weighing process. Even in such a case, the volume of defective products to be shipped out with inaccurate marks of weights can be restrained in various ways(see, Sandoh and Igaki 2001, Sandoh and Igaki 2003, Sandoh and Nakagawa 2003).

In this study, we consider two types of inspection-warranty policy to make a comparison between them. The comparison is carried out through a Stackelberg game formulation to take into account both the consumer's viewpoint and the manufacturer's one.

2 Assumptions and notations

We make the following assumptions: (1) We consider a monopoly. (2) The manufacture weighs each product using a scale and ship out each product after he puts a label on each individual product to show its weight. (3) There are many products to be weighed and therefore we regard the volume of products to be weighed as continuous. The unit of time is defined as the time required for weighing a unit of product. (4) We call the products which are weighed by an uncalibrated scale to be shipped out *defective* regardless of their quality. (5) The scale is inspected at $iT(i = 1, 2, \cdots)$. (6) Inspection activities involve adjustment operation and hence the scale becomes calibrated immediately after inspection. (7) Let c_0 and c_1 respectively express the cost per inspection activity and the cost for weighing a unit of product. (8) For $i = 1, 2, \cdots$, let us denote, by a random variable X_i, the time for a scale to be uncalibrated on an interval $((i-1)T, iT]$. Let $X_1, X_2, \cdots$ be independent and identically distributed with distribution function F and density function f. In addition, we assume that $E[X_i] = \mu << +\infty$. (9) The raw price of the product is given by a. (10) The consumer's revenue by purchasing the product is given by R.

Based on the above assumptions and notations, we consider the following two types of inspection-warranty policy:

[Policy 1]

Each product is shipped out immediately after it is weighed. In this case, we have a risk to ship out defective products, and hence, we devote c_2 to the warranty for the consumer who purchased a defective product. When the consumer purchased a defective product, he/she can receive $W = \alpha c_2 (\alpha > 0)$ through the warranty service. The products shipped out under this policy are called *Type 1 products*. Type 1 product is sold at price $P_1 (< R)$.

[Policy 2]

Products are not shipped out until we assure that the scale is calibrated by inspection. In case the scale is found to be uncalibrated by an inspection activity, all the products waiting for being shipped out are weighed again until the scale is inspected to be normal. The products shipped out under this policy are called *Type 2 products*. The price of Type 2 product is denoted by $P_2 (\geq P_1)$.

Under Policy 2, we never ship out defective products, and therefore we provide the consumer with no warranty on weight-quality. It should, however, be noted that we need secure some space for the weighed products to wait for being shipped out. Let c_3 and c_4, respectively, express the cost for a unit of weighed product to occupy the space per unit of time and the cost for each weighed product to waste a unit of time without being shipped out.

It is very difficult to analytically compare Policy 1 with Policy 2 based on the cost for inspection-warranty policies from the manufacturer's point of view. In the following, we introduce a Stackelberg game formulation to make a comparison between the two policies taking into account the consumer's and the manufacturer's viewpoint.

3 Consumer's optimal reaction

If the consumer purchases a Type 1 product, his expected profit becomes

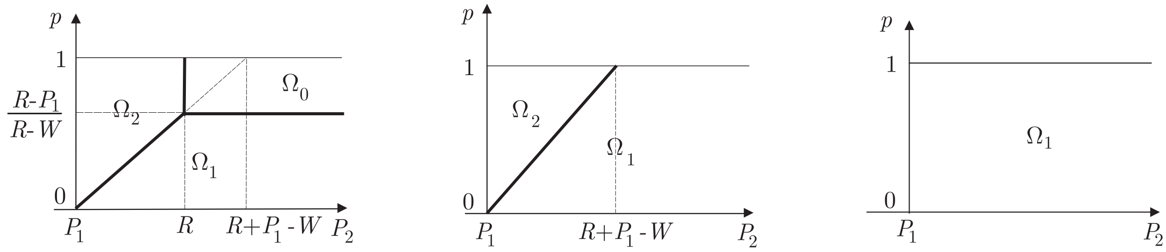
$$\Pi_1(p) = (R - P_1)(1 - p) + (W - P_1)p, \tag{1}$$

where $p = D(T) \equiv \int_0^T F(x)dx/T$. When he chooses a Type 2 product, his expected profit is given by

$$\Pi_2(P_2) = R - P_2, \tag{2}$$

while his expected profit becomes $\Pi_0 = 0$ when he purchases no product.

By comparing $\Pi_1(p)$ with $\Pi_2(P_2)$ or Π_0, we can obtain the optimal reaction by the consumer as depicted in Fig. 1, where $\Omega_i (i = 0, 1, 2)$ in Fig. 1 signifies that the consumer would purchase a Type i product and that purchasing a Type 0 product corresponds to purchasing no product.



(a) $W_0 < P_1$ (b) $P_1 \leq W_0 < R$ (c) $W_0 \geq R$

Figure 1: Optimal reaction of consumer.

4 Manufacturer's optimal strategy

This section first formulates the expected cost per unit of time under each inspection-warranty policy from the manufacturer's viewpoint, and second, discusses an optimal strategy for the manufacturer, considering the consumer's optimal reaction we have observed above.

4.1 Optimal policies

From the renewal reward theory(see, Ross 1970), the expected profit per unit of time under Policy 1 is expressed by

$$Q_1(T) = P_1 - a - \left[c_1 + \frac{c_0 + c_2 \int_0^T F(x)dx}{T} \right], \tag{3}$$

where the manufacturer can control fraction defective p through the inspection time interval T and also the warranty W via c_2 of Type 1 product.

Under Policy 1, we have the following theorem:

Theorem 1
(1) *If $W \geq P_1$, the optimal policy becomes:*

i. *In the case of $\mu > c_0/c_2$, there exists a unique finite optimal inspection time interval $T = T_1^*$, which satisfies*

$$Q_1(T_1^*) = P_1 - a - c_1 - c_2 F(T_1^*), \quad p^* = D_1(T_1^*) = \frac{\int_0^{T_1^*} F(x)dx}{T_1^*}. \tag{4}$$

ii. *In the case of $\mu \leq c_0/c_2$, we have $T_1^* \to +\infty$ with*

$$Q_1(T_1^*) = P_1 - a - c_1 - c_2, \quad p^* = D_1(T_1^*) = 1. \tag{5}$$

(2) *If we have $W < P_1$, let*

$$L(T) = TF(T) - \int_0^T F(x)dx = \int_0^T \overline{F}(x)dx - T\overline{F}(T), \tag{6}$$

then optimal policy becomes:

i. *In the case of $L\{D^{-1}[(R-P_1)/(R-W)]\} > c_0/c_2$, there exists a unique finite optimal inspection time interval $T = T_1^*$ under Policy 1, which satisfies Eq. (4).*

ii. *If $L\{D^{-1}[(R-P_1)/(R-W)]\} \leq c_0/c_2$, we have $T = T_1^* = D^{-1}[(R-P_1)/(R-W)]$ along with $p^* = D(T_1^*) = (R-P_1)/(R-W)$.*

On the other hand, the expected cost per unit of time under Policy 2 becomes

$$Q_2(T,P_2) = P_2 - a - \frac{c_0 + c_1 T + c_3 T^2 + \dfrac{c_4 T^2[1+F(T)]}{2}}{T\overline{F}(T)}, \tag{7}$$

where the manufacturer can control his expected profit through the inspection time interval T as well as the price P_2 of Type 2 product. In addition, it should be noted that $Q_2(T,P_2)$ is increasing in P_2. Under Policy 2, we can show that there exists at least one finite optimal inspection time interval $T = T_2^*$ for a fixed P_2.

4.2 Optimal strategy for the manufacturer

In the following we confine ourselves into the case of $Q_1(T_1^*) > 0$, and then the above observations reveal that the manufacturer's optimal strategy becomes:

(1) If $Q_1(T_1^*) \geq \lim_{P_2 \to R-0} Q_2(T_2^*, P_2)$ for $W < P_1$ or if $Q_1(T_1^*) \geq \lim_{P_2 \to (R+P_1-W)-0} Q_2(T_2^*, P_2)$ for $P_1 \leq W < R$, then the manufacturer's optimal strategy is expressed as

$$T = T_1^*, \quad p = D(T_1^*), \tag{8}$$

under Policy 1 and

$$P_2 > P_1 + (R-W)D(T_1^*), \tag{9}$$

under Policy 2. In this case, the consumer purchases a Type 1 product and the manufacturer's expected profit becomes $Q_1(T_1^*)$.

(2) If $Q_1(T_1^*) < \lim_{P_2 \to R-0} Q_2(T_2^*, P_2)$ for $W < P_1$ or if $Q_1(T_1^*) < \lim_{P_2 \to (R+P_1-W)-0} Q_2(T_2^*, P_2)$ for $P_1 \leq W < R$, then his optimal strategy is given by

$$p \begin{cases} > \frac{R-P_1}{R-W}, & W < P_1 \\ \to 1-0, & W \geq P_1 \end{cases}, \tag{10}$$

under Policy 1 and

$$T = T_2^*, \quad P_2 \to \begin{cases} R-0, & W < P_1 \\ R+P_1-W-0, & W \geq P_1 \end{cases}, \tag{11}$$

under Policy 2. In this case, the consumer purchases a Type 2 product and the manufacturer's expected profit is given by

$$Q_2(T_2^*, P_2^*) = \begin{cases} \lim_{P_2 \to R-0} Q_2(T_2^*, P_2), & W < P_1 \\ \lim_{P_2 \to (R+P_1-W)-0} Q_2(T_2^*, P_2), & W \geq P_1 \end{cases}. \tag{12}$$

(3) If $W > R$, the consumer never pays attention to Type 2. Hence, the manufacturer's optimal strategy becomes

$$T = T_1^*, \quad p = D(T_1^*), \tag{13}$$

under Policy 1 and P_2 set to an arbitrary value under Policy 2 on the condition that $P_2 > P_1$. The manufacturer's expected profit is given by $Q_1(T_1^*)$.

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